MULTIFRACTAL FORMALISM FOR REMOTE SENSING: A CONTRIBUTION TO THE DESCRIPTION AND THE UNDERSTANDING OF METEOROLOGICAL PHENOMENA IN SATELLITE IMAGES

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The ability of fractals to mimic Nature has led to the widespread acceptance of fractal, and, beyond, multifractal models. Such models are in the roots of new approaches in environmental sciences for processing images displaying turbulent-like systems. This paper addresses the problem of detecting critical areas associated with convection in satellite meteorological images. The technique we propose takes information about the spatial domain and relies on general statistical concepts. Due to the turbulent character of the observed atmospheric systems, the multifractal approach is naturaly adopted herein to describe not only the geometrical properties of images but also the underlying physical phenomena involved. The multifractal formalism leads first to the classification of different chaotic parts of systems according to their dynamical significance. It is further exploited to extract information about the places at which convection takes place in the flow. It is shown that it finally allows the determination of information that would be otherwise hidden. Without any temporal information, this remote sensing technique has potential application to infer the convective-scale processes occurring in individual convective systems. More generally, it leads to new insights into the analysis of natural phenomena from still images.

1. Introduction

It is now recognized that many processes in nature, engineering, science and economics exhibit complex scaling behavior [15]: some obvious examples are given by oceanographic and atmospheric phenomena [6,13], other less evidently related systems

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are finance markets [16], topographic landscapes [11] or even heartbeat rythms [12]. This complexity in scaling is commonly reflected as a nonlinear irregular variability of patterns fluctuations across a broad range of scales [22]. Theoretical arguments have shown that turbulence is representative of such processes [2,7]. This has led to a growing interest in obtaining compact description and accurate prediction tools for turbulent-like systems. However, the analysis of turbulent experimental data is still regarded as strongly challenging: classical descriptions are based on Navier-Stokes's or other chaos-spawning differential equations, for which inverse and direct problems are extremely demanding; such approaches do not supply with a satisfactory quantification of the nonlinear variability in multiple scaling [15]. To overcome these limitations, the concept of *Multifractals* has been developed to provide a mathematical model that embraces the irregularities that are, more generally, generic to the evolution of dynamical systems in Nature [15,18]. The multifractal formalism mainly assumes that local singularities exist within the systems and are arranged in a complicated mesh of different scale-invariant (fractal) structures. For a detailed discussion on the multifractal nature of fluid turbulence (and also, more specifically, atmospheric flows), the reader is refered to [6,7,20,21].

In this context, quantification of atmospheric flow patterns as recorded by meteorological parameters such as temperature, wind speed, ..., besides helping prediction of weather and climate, also provides a model for turbulent fluid flows. Namely, climate and other apparently chaotic phenomena can be modeled and even predicted with (multi)fractal methods [6,20]. In particular, in the latest years, the improvement of the technology for observing the Earth from the space has given rise to a growing interest on the developpement of new signal processing tools derived from thermodynamical concepts and their application on satellite images [3,5]. Indeed, for most of the large-scale processes, statistics can only be obtained from satellite observation - and especially infrared (IR) images - because only this kind of data can cover the range of time and space scales involved in atmospheric phenomena [1,10]. The objective of the present work is to show how the multifractal scaling formalism can be: *(i)* applied to process satellite images in order to extract meaningful meteorological patterns; *(ii)* further extended in order to give an interpretation to these patterns with regard to the underlying geophysical phenomena. We will precisely focus on the characterization of the dynamical evolution of convective clouds systems in satellite IR images. The knowledge of the dynamics of convective systems (CS) is of great interest for a number of climate applications - such as general circulation models - as they are mainly responsible for hard weather situations like rainfalls and thunderstorms [1]. However, the analysis of their life-cycle is particularly challenging due to their multifractal, intermittent structure: evidence of scaling in space and time of clouds has been demonstrated in many previous studies [5,9,13]. Multifractal geometry appears naturally as the suitable approach to perform such analysis, as we expect any quantity defined on the atmospheric flow to behave in an ergodic way, and even more to define a multifractal structure [2,21]; indeed, it has already provided various tools for the investigation of clouds properties. In [13], the multifractal cascade phenomenology is used to classify measured rain fields with respect to the nature of the observed clouds. Multifractal temporal fluctuations associated with selfsimilar multifractal spatial patterns for rain areas detection were also considered in [20]. In [8], the main fronts of CS are extracted thanks to a multifractal analysis of singularities in IR images. In [3], a multifractal model is used to analyze the textural roughness properties of different types of clouds. In [5], the authors use multifractal cascade models to study properties of CS and to identify convective-scale processes therein. Recently, a new method for the assessment and tracking of pluviometry in CS in still IR images has been proposed [24]. Following [8], the main underlying idea is that strong transitions in the signal are related with the dynamics of the flow. The streamlines of IR data are first identified by a multifractal singularity analysis; a proxy image that simulates pure horizontal advection is then derived from these streamlines; finally, from the comparison of both original and proxy images, convective areas of the flow are localized, and further identified with purely raining places. It appears from this study that the analysis of the flow can be reduced to the characterization of some singular points derived from the multifractal formalism. In this paper, basing on the assertion that forms visualized in still images account for the underlying motion, we develop and evaluate this approach to the problem of motion characterization. We show that the formalism derived from the multifractal model of [24] allows to identify the "hidden sources" for the process, providing critical information about the flow evolution and the nature of the underlying motion: these sources correspond to areas where the (2D) image motion field is divergent, or, equivalently, where the (3D) flow is convective.

The remainder of the paper is organized as follows. In section 2, we review the multifractal model that enables the extraction of meaningful patterns in the IR images and the reconstruction scheme derived from this model. In section 3, we relate this scheme with the hypothesis describing advective motion and we develop the formalism of the sources. In section 4, we compare the results with a physically-based optical-flow model and we provide an interpretation to the sources regarding the underlying phenomena. As a conclusion, we give the natural continuation to bring to this work for further validation/use of the model.

2. Multifractality help to decipher complex image patterns

Multifractality is a property of turbulent-like signals which is present in very different physical systems [7]. It is generally reported on intensive, scalar variables of chaotic structures. In the following, we present one of the possible formalisms used to characterize multifractality, that was introduced in [25], and apply it on IR images.

The multifractal structure of a signal I can be assessed by defining at any particular point $\vec{x}$ a measure μ in the way [25,26]:

$$\mu(B_r(\vec{x})) \;\equiv\; \int_{B_r(\vec{x})} d\vec{y}\, |\nabla I|(\vec{y})\,, \tag{1}$$

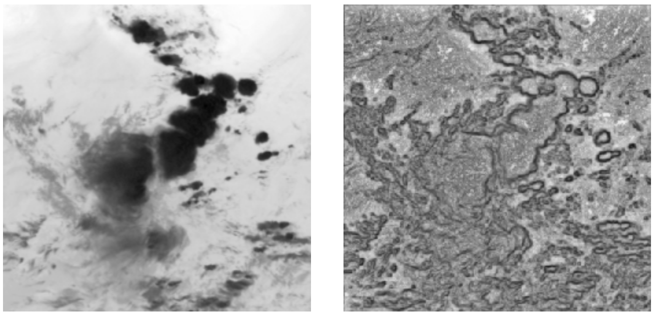

Figure 1.: Left: IR image acquired by the geostationnary satellite Meteosat displaying a typical CS. Right: representation of singularity exponents; the darker the pixel, the lower the value of this exponent at that point. Lower values (*i.e.* most singular points) are found near the boundaries.

where the norm of the gradient $|\nabla I|$ is weighted over all the points in the ball $B_r(\vec{x})$ of radius r around $\vec{x}$, and observing local power-law scalings [a]:

$$\mu(B_r(\vec{x})) \approx \alpha(\vec{x})\, r^{h(\vec{x})}. \tag{2}$$

That power-law behaviour implies that there is no privileged scale of observation: the signal is self-similar as all the dependence in the scale r is contained in the factor $r^{h(\vec{x})}$, not $\alpha(\vec{x})$. Let then F_{h_0} be the level-set associated to a given value h_0:

$$F_{h_0} \equiv \{\vec{x} : h(\vec{x}) = h_0\}, \tag{3}$$

hence, all points $\vec{x}$ in the space are arranged according to the local *singularity exponent* $h(\vec{x})$ they were assigned. The family of subsets F_h are of fractal character (*i.e.* they exhibit the same geometrical structure at different scales) due to the scale invariance of the function $h(\vec{x})$ and provide a multifractal splitting of the image. For a full discussion, the reader is refered to [26] and all references therein. Besides, this decomposition has a strong dynamical meaning: for instance, one can deduce the statistics of changes in scale just knowing the dimensions of the fractal components [18]. Let also notice that, in turbulence theory, an expression similar to eq. (1) defines the local dissipation of energy in the flow; thus, the decomposition F_h can be regarded as the geometrical representation of the multifractal cascade of energy between the different scales [2,9]. However, the current approach does not depend on the specifics of the cascade phenomenology, the geometrical attributes of the multifractal hierarchy are analyzed directly.

In the context of image processing, the point $\vec{x}$ refers to the pixel, the variable $I(\vec{x})$ is the graylevel value and the measure μ expresses the local variability of graylevels around $\vec{x}$. Performing a multifractal decomposition with eq. (2) finally

[a]Note that the exponents $h(\vec{x})$ in eq. (2) could be rewritten $h(\vec{x}) + d$ where d is the dimension of the space so that they will be the same for measures of the same degree of regularity disregarding the dimensionality [26].

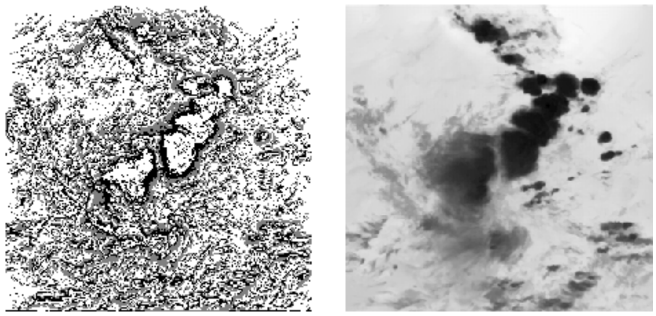

Figure 2.: Left: MSM estimated on the sample IR image of Fig. 1; pixels of the MSM are assigned a non-white graylevel according to the orientation of the gradient over this set (grey: negative; black: positive). Right: reconstruction out of the data defined on the orientated MSM.

amounts to locally quantify the strenght of the transition the signal undergoes around each pixel. In practice [26], to overcome the difficulties due to the image discretization, the gradient $I(\vec{x})$ is simply estimated by finite difference at the distance of one resolution box (one pixel)[b]. The singularity exponents are computed through a log-log linear regression on the wavelet transform [14] of the measure, which has been proven to also exhibit multifractal behaviour [26], and not of the measure itself. When considering meteorological IR images (Fig. 1, left), the graylevel intensity is identified with thermal IR radiance, measured on higher layers of the atmosphere, which is a typical turbulent variable following a complicated pattern. Namely, it was shown in [8] that IR images are of multifractal nature. In Fig. 1, right, we can see a typical segmentation induced by the spatial distribution of the singularity exponents. The F_h splitting consists of an edge-like most singular manifold surrounded by manifolds of decreasing singularity as we move away from it [26].

In the multifractal decomposition, one set, called the *Most Singular Manifold* (MSM), is of particular importance: the manifold F_∞ associated to the minimum value for the singularities $h_\infty \leq h(\vec{x})\ \forall\, \vec{x}$, whose existence is ensured by the multifractal theory [25]. The MSM has usually a fractal co-dimension of 1 and resembles the edges or contours of the objects in the case of natural images [26] or relevant dynamical points in other systems [8]. In Fig. 2, left, the MSM gathers the pixels where the discontinuities in graylevel are the more appreciable, thermal fronts indeed. The MSM is known as the most informative set in the image [8], but its most relevant property is found in its interpretation as the origin of the multifractal hierarchy [23]. It is possible (under certain statistical hypothesis) to reconstruct the multifractal completely from the values of the gradient restricted to the MSM, ∇I_{h_∞}:

$$I(\vec{x}) \ = \ \vec{g} \otimes \nabla I_{h_\infty}(\vec{x}) \tag{4}$$

[b] This definition is derived from the linear increment formalism originally employed for the analysis of multifractal behavior in turbulent flows [7].

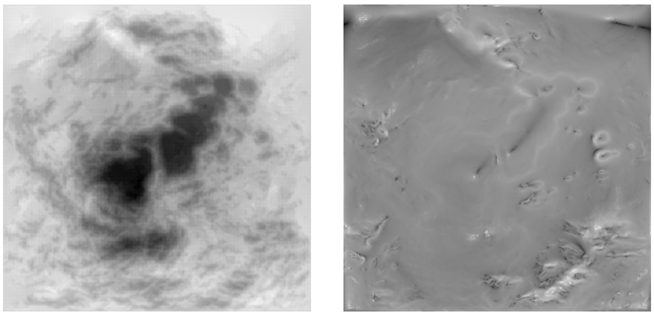

Figure 3.: Left: reduced image computed from the orientated MSM in Fig. 2. Right: representation of the norm of the source field; gray pixels in the sources stand for values close to the average, while zeros are represented by black pixels and poles by white pixels.

with a particular reconstructing kernel $\vec{g}$ defined in Fourier space (see Fig. 2, right).

3. Multifractal sources provide relevant motion information

When dealing with large-scale phenomena in the atmosphere (scales of a few tens of km or more), one usually appreciates several elements that configure the dynamics: almost 2D motion, vertical stratification and also a strong influence of Earth's rotation. Indeed, transport in the atmosphere is dominated by advection, and is quasi-horizontal. However, there are a number of different physical mechanisms by which vertical transport may be present [1]. In this context, we will see that the multifractal formalism introduced above can be further completed in order to extract the places where the flow is not contained in the surface of the image.

Indeed, it is possible to obtain more information from the MSM than just reconstructing the signal from it. The detection of the MSM constitutes the stumbling block to infer the properties of the local motion in the flow. In [24], it was shown that detecting the MSM is equivalent to detect the main instantaneous (2D planar projection of the) streamlines of the flow. The underlying hypothesis is that vertical movements (mainly inside clouds) modify the spatial variability of temperature [10]. More precisely, as the MSM consists of flow lines, the situation where the flow is purely advective correspond to the situation where the image gradient is perpendicular to the MSM lines with constant modulus. Namely, when temperature is advected by the flow, it is diffused horizontally, i.e. inside planes parallel to the image surface and it is possible to track its movement; however, when temperature is convected, there is a temperature flow transversal, at an unknown rate, to the plane of image acquisition. Thus, for any scalar quantity advected in a region, its average across that region is conserved; more mathematically, the gradient is perpendicular to the streamlines, and with constant modulus. These observations lead to the concept of a *Reduced image R*: in eq. (4), the actual gradient over the MSM ∇I_{h_∞} is replaced with a vector field consistent with the advection hypothesis *i.e.* perpendicular to the MSM and of constant modulus (and with, by convention,

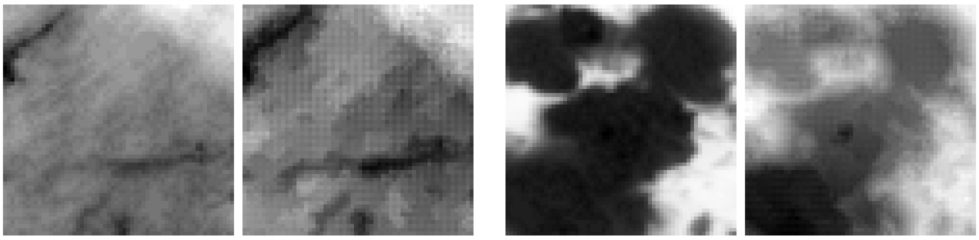

Figure 4.: Comparison of original IR image and its reduced counterpart on two excerpts extracted from Fig. 1 and 3 *resp.* Left: the flow is dominated by advection, the images are rather similar. Right: convective activity inside the CS is present, the images are very different. The range of graylevels was expanded in order to show as many details as possible.

the same orientation as the original gradient) [8,24]. Comparing original and proxy signals, we can deduce the temporal evolution of the flow. A simple way to assess if the advective assumption holds in a given area is to verify if the gradient along the MSM is constant and perpendicular to the MSM, *i.e.* if I and R coincide in this area. Where I differs from R, convective movements (perpendicular to the plane of the image) may be present (see Fig. 4).

In order to propose a more quantitative criterion, we use the formalism of sources previously introduced in [24]. Let first define the vectorial measure $\vec{\mu}_I$ associated to the signal I as follows. For any subset (region in the image) A, the vectorial measure $\vec{\mu}_I(A)$ is given by:

$$\vec{\mu}_I(A) \;\equiv\; \int_A d\vec{y}\, \nabla I(\vec{y}) \tag{5}$$

and an analogous definition for the measure $\vec{\mu}_R$ associated to the reduced signal R. The absolute variations [19] of both $\vec{\mu}_I$ and $\vec{\mu}_R$ are multifractal (in the sense of eq. (2)) with the same singularity exponents $h(\vec{x})$, that is [c]:

$$|\vec{\mu}_I|(B_r(\vec{x})) \sim |\vec{\mu}_R|(B_r(\vec{x})) \sim r^{h(\vec{x})}\,. \tag{6}$$

Consequently the two measures are absolutely continuous with respect to each other [19], and for that reason one measure can be represented as the integral with the other measure of an appropriate vectorial density $\vec{S}$. We define the vectorial *field of Sources* as the Radon-Nykodin derivative [19]:

$$\vec{S}(\vec{x}) \;\equiv\; \lim_{r \to 0} \frac{\vec{\mu}_I(B_r(\vec{x}))}{\vec{\mu}_R(B_r(\vec{x}))} \tag{7}$$

where the limit of the ratio is taken at decreasing scales r. Technically, we make use of the commutative algebra of complex numbers to compute $\vec{S}$: the 2D vector fields defined through eq. (5) are seen as complex fields and the vectorial division in eq. (7)

[c] In theory, R attains the most uniform intensity distribution compatible with the multifractal structure of I.

is assumed in the complex plane. The sources are not of multifractal character, as the singularities of I have been removed by dividing by those of R; as a consequence, they are displaying a completely different, more regular structure (Fig. 3, right). It was shown in [24] that they enable to easily characterize advection and convection, avoiding the need of considering regions separately and without having to process a sequence. Over advective-dominated areas $\vec{S}$ will be a constant, as the image and its reduced counterpart coincide. On the contrary, on areas dominated by vertical transport, the variations of $\vec{S}$ will be large. Taking into account that roughly $\nabla I \approx \vec{S}\ \nabla R$ (the sources represent the matrix field of transformation between the gradients of I and R), advection means that $\vec{S}$ is a constant, real number. For convection, on the contrary, the highest discrepancy between ∇I and ∇R will happen when one of them vanishes while the other is still finite; hence, the strongest convection appears in the zeroes and poles of $\vec{S}$.

4. Validation and comparison with temporal information

In order to validate the role of the sources, we present evidence on the connection of sources with the dynamics of the atmospheric flow. For that purpose, we compare the features extracted through the estimation of the source field with the description of motion obtained through velocity fields.

Fig. 5 shows two IR samples belonging to a common sequence of images displaying cloudy systems that are known to involve convective activity, namely CS [1]. Hence, some temporal information can be inferred. The most common method for meteorological sequence analysis consists in estimating the apparent motion through dense velocity fields computation. In particular, using optical flow (OF) methodologies [4], it is possible to assign a velocity vector to each point in the image using conservation hypothesis and regularisation constraints. Even if these hypotheses are seldom satisfied in the case of natural fluid flows, OF may provide a reasonable approximation of apparent motion. If we compute the divergence of the resulting flow, we observe (see Fig. 5) a very good correspondence of large divergence values with singular focii in the vector field of sources. The sources correspond to the purely convective area of CS, dominated by vertical motions [24]: singularities of the motion field (that can be connected to convection) are related to the spatial distribution of flow transport across scales. This result is quite promising: sources are computed using one image only, contrary to OF that requires sequence of images, and provide however relevant, comparable information about the temporal evolution of CS.

5. Conclusion

In this paper, we have developed and validated a multifractal technique to detect and locate critical areas of meteorological structures, associated with 2D divergent motion or, equivalently, 3D vertical transport, that are displayed in satellite IR images. Due to the fact that the evolution of atmosphere involves fluid phenomena, the multifractal formalism, derived from thermodynamical concepts, appears as the most suitable approach to analyze such data.

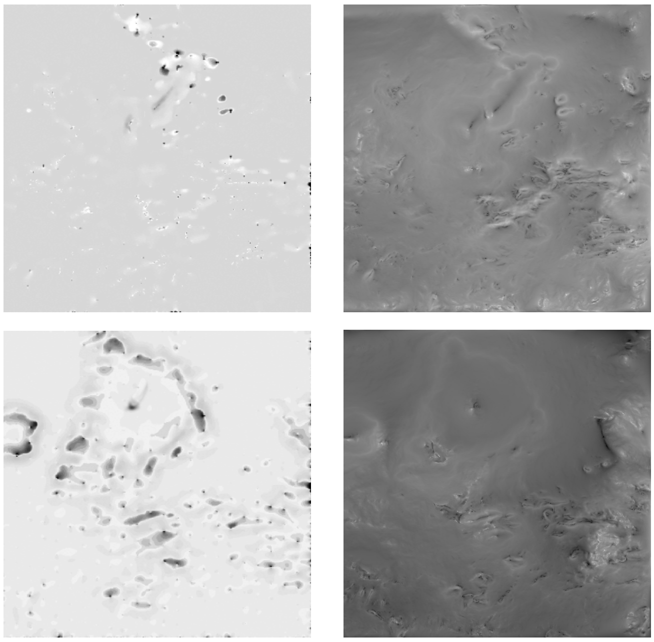

Figure 5.: Comparison of the information provided by velocity field estimation and source field computation on IR images. Selected samples were extracted from a common sequence displaying the temporal evolution of the CS of Fig.1. Left: divergence of the velocity fields estimated with OF approach; non null divergent areas are represented in black (positive) and white (negative); average gray values correspond to areas with (close to) null divergence. Right: norm of the sources.

The main result of this study regards the characterization of the, so-called, source field and its specificity. Sources reveal the gradual deviation from advection to convection because the multifractal structure is completely determined by the properties of the MSM. Once we know how to translate any constraint on the dynamics (as for instance advection) to the MSM, we can construct a reduced signal verifying that constraint, and then compare it with the original signal. The Radon-Nikodym derivative will remove the part that both signals share (multifractal structure), highliting their differences (dynamics). The multifractal scheme proposed does not require of any continuity or smoothness hypothesis (as OF methods, for instance) and offers an instantaneous information derived from still images (in opposition to OF, that require a sequence of images sampled at a rate fast enough). The power of multifractal analysis lies in the existence of a hierarchy which is conserved by the dynamics and which can then be used to know more about it. Besides, any other problem in which a multifractal structure has been reported is a good candidate for a multifractal analysis of this kind. Sources can be calculated in order to characterize the underlying dynamical properties. More generally, they provide a new way to analyze and interpret many natural systems.

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